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Ground state of a Bose-Einstein condensate which scatters coherently laser radiation

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Abstract. We present a self-consistent method of taking into account back action of a laser radiation to a Bose-Einstein condensate of neutral atoms. The light is coherently scattered inside the degenerate atomic sample, thus its intensity and, consequently, the atomic ground level AC Stark shift are spatially varying. This leads to a small deformation of the atomic cloud and, if the external radiation is abruptly switched off, to generation of collective excitations.

PACS. 03.75.Fi Phase coherent atomic ensembles; quantum condensation phenomena – 32.80.-t Photon interactions with atoms – 42.50.Vk Mechanical effects of light on atoms, molecules, electrons, and ions

1 Introduction

Bose-Einstein condensation in dilute atomic vapour has become, since its first experimental demonstration in 1995 [1–3], an important subject of modern physical studies. It is an unique possibility to get an ensemble of nearly 10^6 magnetically trapped atoms all being in the same quantum state of center-of-mass motion. Such a medium is a degenerate system of weakly interacting particles which behaves as a quantum fluid possessing many spectacular features. It is very important that a straightforward theoretical analysis of various properties of a Bose-Einstein condensate (BEC) of neutral atoms is possible, and the predictions can be tested experimentally.

The most informative methods of experimental study of BEC of neutral atoms are laser spectroscopic ones. During the very beginning of these researches an absorption (destructive imaging) was used [1,4]. However, there is a possibility of a non-destructive testing of a BEC sample by measuring a position-dependent phase shift of a far-resonant laser light transmitted without significant absorption. Such a method was adapted to BEC diagnostics in reference [5], and its further applications include production of impressive images of sound waves in a BEC [6], precise determination of an atomic cloud shape [7] demonstrating an excellent agreement with the mean-field theory of a BEC ground state [8] and providing a significant improvement in experimental determination of the s-wave scattering length of sodium atoms, and detection of Bose-Einstein condensation in a vapour of ^{7}Li, an element with a negative scattering length which admits a metastable BEC existence only if the total number of atoms in the sample is less than 1300 [9].

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From the point of view of spectroscopy, a total elimination of inhomogeneous broadening is very interesting. The recent work on slowing the group speed of light in such a degenerate atomic ensemble down to 17 m/s by means of electromagnetically induced transparency [10] has stressed once again new prospectives for atomic physics and nonlinear optics opened due to novel achievements both in theory and experiment dealing with ultracold atoms.

It is widely believed that back action of laser light to BEC can be neglected provided that the number of absorbed photons is much less than the total number N of atoms and the angle of coherent scattering and, hence, the kinetic momentum transferred from a photon to an atom are small. The measurements of reference [9] made using relatively large laser intensity (250 mW/cm^2) and moderate detuning (between 20 and 40 radiative halfwidths of the absorption line) were an example of obvious violation of the latter condition and, therefore, significant heating of atoms after a single shot of the probe laser.

The subject of the present paper is to estimate a back action of laser radiation to a BEC in a case when these effects only modify but do not destroy degeneracy of an ultracold ensemble of bosonic atoms in a magnetic trap.

2 Basic equations

The existing theories of light coherent scattering by a BEC of neutral atoms assume that the BEC ground state is not changed by an external radiation [11,12]. This assumption simplifies the analysis very much and seems to be very natural. Indeed, an atom excited by the incident electromagnetic wave ($k_\mathrm{L} = \omega_\mathrm{L}/c$ is the wavenumber, c is the speed of light) preferably emits a photon into a small body angle of order of $(k_\mathrm{L} R)^{-1} \ll 1$, where R is the condensate size, and returns to the condensate because the probability of such a process is enhanced in proportion to the total number of atoms $N \gg (k_\mathrm{L}R)^2$ by the Bose-Einstein statistics. Only very small fraction (approximately $(k_\mathrm{L}R)^2/N$) of photons is scattered incoherently, so atomic sample heating and subsequent decrease of number of atoms in condensate are quite slow processes.

Nevertheless, there is a cause of modification of the macroscopic wave function of atoms composing a BEC under action of laser radiation tuned from resonance far enough to prevent significant absorption. Namely, if the light is refracted inside the atomic cloud then its intensity varies in space. This means that the AC Stark shift of the atomic ground state is also spatially non-uniform and acts as an addend to the trap potential. The value of AC Stark shift can be expressed easily *via* the local light intensity I, the laser detuning $\varDelta$, the halfwidth γ of the incoherent scattering line, and the photon absorption cross-section $\sigma \approx 2\pi k_\mathrm{L}^{-2}$ at the line center. This term should be added to the set of equations of quantum hydrodynamics of a BEC (see, *e.g.*, the review [13] and references therein) which is equivalent to the Gross-Pitaevsky equation for the BEC macroscopic wavefunction $\varPsi = \sqrt{n}\exp(\mathrm{i}\phi)$, where n is the condensate local density, the phase ϕ is related to the hydrodynamic velocity $\mathbf{v}$ as $\nabla\phi = M\mathbf{v}$, M is the atomic mass. We use the system of units where

$$\hbar = 1,$$

i.e. express energy in inverse seconds. Thus

$$\frac{\partial}{\partial t}n + \nabla(n\mathbf{v}) = 0, \tag{1}$$

$$M\frac{\partial}{\partial t}\mathbf{v}+\nabla\left(\frac{M}{2}v^2-\frac{1}{2M\sqrt{n}}\nabla^2\sqrt{n}+U+gn+\frac{I\sigma\gamma}{2\omega_\mathrm{L}\varDelta}\right)=0. \tag{2}$$

Here the constant $g = 4\pi a/M$ characterizes the interaction between atoms, $a > 0$ is the s-wave scattering length (in the present paper only a repulsive interaction case is considered). The harmonic trap potential is assumed to be cylindrically symmetric,

$$U = \frac{M}{2}\omega_\mathrm{tr}^2\left[r^2 + (z/\varLambda)^2\right], \tag{3}$$

where $r = \sqrt{x^2+y^2}$ is the transverse radial co-ordinate, $\varLambda$ is the dimensionless aspect ratio.

The set of equations (1, 2) should be supplied by the equation of electromagnetic wave propagation. We denote the complex electric field value by $\tilde{E}$, so $I = (c/8\pi)|\tilde{E}|^2$. To describe its variation in space and time, we apply the same equation as that of reference [12], where it serves a starting point for development of a generalized diffraction theory with respect to optical properties of BEC:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\tilde{E} = \frac{\gamma}{\varDelta}k_\mathrm{L}\sigma n\tilde{E}. \tag{4}$$

Our approach is, in fact, a next step in the theoretical research direction outlined by reference [12]. Equation (4) not only yields an asymptotic solution describing the scattered wave far from the BEC but also can be used for evaluation of light field parameters inside the atomic cloud with the local density n.

Let us consider a stationary solution for a trapped atomic BEC interacting with a monochromatic radiation $\tilde{E} = E\exp(\mathrm{i}\omega_\mathrm{L}t)$, where E is a time-independent complex amplitude. Thus, $\mathbf{v}$=0 and n does not depend on t, the set of equations (1, 2, 4) reduces to

$$\tilde{\mu} = -\frac{1}{2M\sqrt{n}}\nabla^2\sqrt{n} + U + gn + \frac{I\sigma\gamma}{2\omega_\mathrm{L}\varDelta}, \tag{5}$$

$$\nabla^2 E + k_\mathrm{L}^2 E = \frac{\gamma}{\varDelta}k_\mathrm{L}\sigma n E. \tag{6}$$

Since the light is scattered by the BEC to small angles, the geometric optics approximation [14,15] is adequate to our problem. It is important to note that this approach works perfectly inside the finite-size atomic cloud. Oppositely, at large distances, a simple picture of light rays those do not cross each other is not valid. However, behaviour of the light in this far zone is irrelevant to our problem of determination of AC Stark shift inside the BEC. We introduce the eikonal function ζ which obeys the following equation:

$$(\nabla\zeta)^2 = 1 - \frac{\gamma\sigma n}{k_\mathrm{L}\varDelta}\cdot \tag{7}$$

Also we define the unit vector $\mathbf{s}$ directed along the light ray passing through a given point of space, $\nabla\zeta = |\nabla\zeta|\,\mathbf{s}$. Then the equation for the light intensity takes a simple form

$$\nabla(I\mathbf{s}) = 0. \tag{8}$$

For a sake of simplicity, we assume that the incident plane light wave (its intensity is denoted by I_in) travels along z, the symmetry axis of the trap.

3 Elimination of the electromagnetic field variables

Further simplifications can be made after one notes that, for the realistic values of $n \sim 10^{12}$ cm^{-3}, $k_\mathrm{L} \sim 10^5$ cm^{-1}, and $|\varDelta| \gg \gamma$ the second term in the right hand side of equation (7) is very small compared to unity. Then we can get that the vector $\mathbf{s}$ nearly coincides with the unit vector $\mathbf{e}_z$ in z-direction,

$$\mathbf{s} = \mathbf{e}_z + \delta\mathbf{s}, \tag{9}$$

and for the difference the following equation holds:

$$\frac{\partial}{\partial z}\delta\mathbf{s} = -\frac{\gamma\sigma}{2k_\mathrm{L}\varDelta}\nabla_\perp n. \tag{10}$$

Here $\nabla_\perp$ is the two-dimensional gradient operator in the perpendicular (x, y) plane. In the linear order with respect to the small parameter $\gamma n k_{\mathrm{L}}^{-3}/\Delta$ we can write $\nabla \mathbf{s} = \nabla_\perp \delta \mathbf{s}$. Then equation (8) can be solved easily:

$$I = I_{\mathrm{in}} \exp\left(\frac{\gamma\sigma}{2k_{\mathrm{L}}\Delta}\, \hat{\Xi}(n) \right). \qquad (11)$$

Here we introduce the operator

$$\hat{\Xi}(n) = \int_{-\infty}^{z} \mathrm{d}z'\, \nabla_\perp^2 \int_{-\infty}^{z'} \mathrm{d}z''\, n(r, z''). \qquad (12)$$

Due to axial symmetry of the system, the two-dimensional transversal Laplacian $\nabla_\perp^2$ reduces to $\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}$.

Now we can substitute equation (11) into equation (5) and thus get an equation containing the unknown function n only. Since the argument of the exponent in equation (11) is small, $|I - I_{\mathrm{in}}| \ll I$, we expand I up to the term linearly proportional n. The constant part of the AC Stark shift is extracted from the chemical potential,

$$\mu = \tilde{\mu} - \frac{I_{\mathrm{in}}\sigma\gamma}{2\omega_{\mathrm{L}}\Delta}\,. \qquad (13)$$

Namely, μ ought to be compared directly to the chemical potential of an atomic Bose-gas in a trap with no external radiation. Finally,

$$\mu = -\frac{1}{2M\sqrt{n}} \nabla^2 \sqrt{n} + U + gn + fg\hat{\Xi}(n). \qquad (14)$$

The dimensionless parameter

$$f = \frac{I_{\mathrm{in}}\sigma^2\gamma^2}{4g\omega_{\mathrm{L}}k_{\mathrm{L}}\Delta^2} \qquad (15)$$

characterizes the relative (compared to repulsive interatomic interaction) contribution of spatially non-uniform light shift to the balance of all the forces determining the equilibrium state of the BEC. It is worth to note that f is always positive for positive g, regardless to the sign of Δ.

Let us make some comments to equations (14, 15) and begin with evaluation of the parameter f. Firstly, writing the Stark shift in the form $(I\sigma\gamma)/(2\omega_{\mathrm{L}}\Delta) = |DE/2|^2/\Delta$, where D is the transition dipole moment and γ is the absorption line halfwidth, one assumes that the laser detuning is of order of Γ or larger. Here $(2\Gamma)^{-1}$ is the lifetime of the excited state of an atom reduced from the value $(2\gamma)^{-1}$ by a factor of about $N(k_{\mathrm{L}}R)^{-2}$ because photon scattering by a BEC is an essentially collective process [11,12] (similarly, such a bosonic enhancement of the forward coherent scattering of photons leads to a small decrease of the incoherent absorption line width 2γ with respect to its usual value $(4/3)D^2k_{\mathrm{L}}^3$; the difference is of order of $\gamma/(k_{\mathrm{L}}R)^2$ [12]). So, in a case when the laser detuning is less than Γ our simple treatment overestimates the AC Stark shift.

Secondly, for realistic experimental conditions, we estimate $\Gamma/\gamma \sim 10^2$. We can represent f as a product of the two quantities, $W = I_{\mathrm{in}}\omega_{\mathrm{L}}^{-1}\sigma(\gamma/\Delta)^2$ and $T_{\mathrm{g}} = \sigma/(4gk_{\mathrm{L}})$. One can find easily that the optical excitation rate W does not exceed the value $(I_{\mathrm{in}}/I_{\mathrm{sat}}) \times 10^3\ \mathrm{s}^{-1}$ where I_{sat} is the saturation intensity of the resonant atomic transition. For the D_1-line of an alkali atom $I_{\mathrm{sat}} \sim 10\ \mathrm{mW/cm}^{-2}$. As follows from s-wave scattering length measurements for Na [7,16] and ^{87}Rb [17], the repulsive interaction constant is of order of $10^{-10}\ \mathrm{cm}^3\,\mathrm{s}^{-1}$. Hence, $T_{\mathrm{g}} \sim 10^{-5}$ s and $f \sim 10^{-2} I_{\mathrm{in}}/I_{\mathrm{sat}}$.

Also we should check validity of the steady-state approach. Indeed, if the laser detuning is too small or the intensity is too high then the photon absorption rate differing from the Stark shift by the factor $\Gamma/|\Delta| \lesssim 1$ may exceed the trap frequency ω_{tr} which determines the rate of hydrodynamic non-stationary motion in the BEC. This means that the BEC is destroyed before its shape changes significantly under the action of the laser radiation. Therefore, we assume hereafter that the restriction

$$f \ll 1 \qquad (16)$$

holds. Equation (16) means, in particular, that the external radiation influence to BEC shape can be neglected if BEC is very dilute, so the mean-field potential gn is small compared to the "quantum pressure" $-(1/2M\sqrt{n})\nabla^2\sqrt{n}$. Oppositely, it makes a sense to study such an influence within the Thomas-Fermi approximation when the "quantum pressure" is negligible. Thus equation (14) takes the following form:

$$\mu = U + gn + fg\hat{\Xi}(n). \qquad (17)$$

4 Results and discussion

Equation (17) admits, most probably, no exact analytic solution. Nevertheless, there is a physically transparent way to solve it by iterations, provided that equation (16) is valid. Namely, we consider the chemical potential μ as a fixed parameter. As an initial approximation we choose the local density n_0 for the BEC in a case of absence of the external radiation:

$$n_0 = \frac{1}{g}(\mu - U), \qquad (18)$$

if $U < \mu$, otherwise $n_0 = 0$. The expression for the number density n_j obtained after the jth iteration is similar to equation (18):

$$n_j = \frac{1}{g}(\mu - U_j), \qquad (19)$$

if $U_j < \mu$, and $n_j = 0$ otherwise. Here the effective potential

$$U_j = U + fg\hat{\Xi}(n_{j-1}) \qquad (20)$$

contains the AC Stark shift calculated using the BEC number density obtained at the previous iteration. Solution of equation (17) by this method was performed numerically. For various values of f after 14 iterations we got $|n_j - n_{j-1}| < 10^{-9} \max(n_j)$ or even better accuracy.

If there were no external radiation, the number of atoms in the BEC would be

$$N_0 = \frac{8\pi \Lambda \mu R^3}{15g} \cdot \tag{21}$$

for a given μ [8]. Here the condensate size R in the transverse direction obeys the formula

$$R = \sqrt{\frac{2\mu}{M\omega_{\rm tr}^2}} \cdot \tag{22}$$

Analysis shows that for non-zero f the actual total number N of atoms in a BEC is larger than N_0 defined for the same μ, and $(N - N_0)/N$ is non-vanishing already in the first order in f. From the other hand, one may investigate an influence of the radiation scattering to the shape of BEC consisting from the given number N of atoms. If the light intensity were constant throughout the atomic cloud the total chemical potential $\tilde{\mu}$ would increase by the value of AC Stark shift, *i.e.* the reduced quantity μ defined by equation (13) would remain unchanged. However, both the light field and the BEC macroscopic wavefunction are allowed to alter in a self-consistent way. It results in a small negative correction to the chemical potential μ compared to a BEC consisting of the same number N of atoms but not irradiated by a laser light.

In Figure 1 the numerical results for different set of parameters are displayed. The horizontal axes correspond to the z and r co-ordinates divided by the characteristic size R defined by equation (22). The vertical axis represent the dimensionless density difference

$$\delta n = \frac{g}{\mu}(n - n'), \tag{23}$$

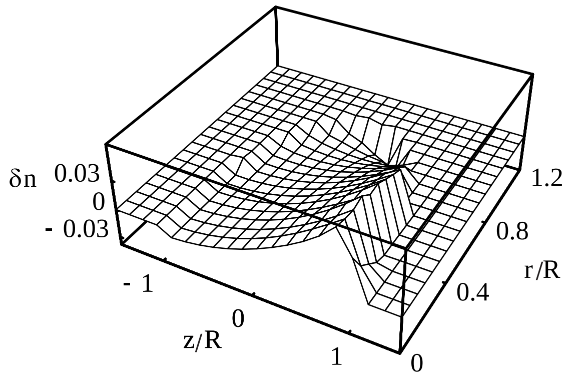
where n' is the density of a BEC which consists of the same number N of atoms but is not irradiated by a laser light. Its chemical potential is therefore larger: $\mu' = \mu(N/N_0)^{2/5}$. So n' is calculated using a formula similar to equation (18) but with μ' placed instead of μ. The dimensionless density difference δn characterizes displacement of the BEC under the action of the laser light compared to its equilibrium state in the absence of external radiation.

One can see from that the displaced fraction of the BEC is of order of $f\Lambda^2$. If the laser is abruptly switched off then, after a transient process of free induction decay has been completed on a time scale of about $\Gamma^{-1} \ll \omega_{\rm tr}^{-1}$, the BEC occurs in a collective excitation state [18] and begins to oscillate. It is interesting to note that a short intense pulse being absorbed in an optically thin BEC causes spatially uniform decrease of the local density which also leads to a BEC collective excitation but of another kind (breathing mode-like instead of a dipole one).

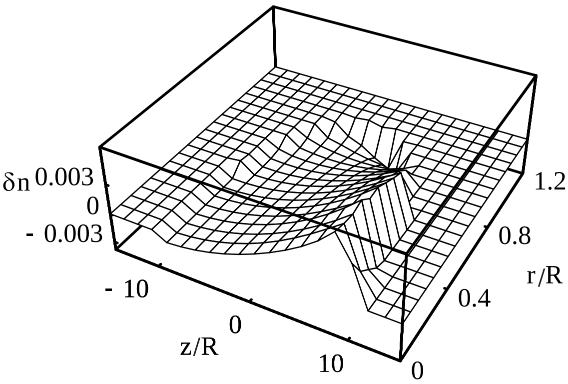
The numerical results illustrate a certain scaling invariance of solution. This property of equation (17) can be easily derived if we take into account a harmonic form of the trap potential U. Namely, if the aspect ratio Λ is changed to Λ_1, f is changed to $f(\Lambda/\Lambda_1)^2$ then, for a fixed μ, N/N_0 remains constant and the number density n (and, hence, δn) scales in proportion to Λ/Λ_1. The ratio N/N_0 is equal to 1.02128 for the set of parameters of Figure 1. For f and Λ being varied so that $f\Lambda^2 = 0.0033$ is kept constant, our numerical simulations give $N/N_0 = 1.00647$. Also our iterative method of solving the equation (17) is found to be divergent for $f\Lambda^2 \gtrsim 0.012$, even in its modified form which uses the following expression for the jth iteration:

$$(1+\alpha)n_j = \alpha n_{j-1} + \frac{1}{g}(\mu - U_j) \tag{24}$$

(here α is a number which can be taken negative as well as positive). It is not clear whether it is simply a disadvantage of the method or it means that solutions for larger values of $f\Lambda^2$ have really new distinctive features.



(a)



(b)

Fig. 1. Dimensionless density difference δn *versus* r, z co-ordinates, (a) $\Lambda = 1, f = 0.01$, (b) $\Lambda = 10, f = 0.0001$.

It is interesting to compare back action to a BEC caused by coherent and incoherent light scattering. The latter process has been recently analysed theoretically [19]. The authors of reference [19] used the paraxial approximation to describe laser radiation propagation in a BEC. From their approach, which includes quantization of the electromagnetic field, a master equation for the bosonic atoms density matrix follows. Coherent light scattering is not considered in reference [19], but all the attention is paid to dissipative processes, namely, to phase diffusion and depletion of a BEC. Since characteristic size of a BEC sample is much larger than the laser radiation wavelength, depletion is the dominant dissipative process. The depletion rate found in reference [19] can be written, in notation used in our paper, as

$$2w_{\mathrm{D}} = \frac{1}{8}\left(\frac{\gamma}{\Delta}\right)^2 \frac{\sigma I}{\omega_{\mathrm{L}}} \tag{25}$$

and represents, in essence, the rate of incoherent scattering of photons by an atom in the case of large detuning. The small factor 1/8 appears, probably, because of the approximation used in reference [19] which considers only paraxial electromagnetic field modes. In any case, this factor is not very significant, and we can estimate, by the order of magnitude, the depletion rate as

$$2w_{\mathrm{D}} \sim fgk_{\mathrm{L}}^3. \tag{26}$$

Typically, $g \sim 10^{-11}$ $\mathrm{s}^{-1}\,\mathrm{cm}^3$, $k_{\mathrm{L}} \sim 10^5$ cm^{-1}. Thus we obtain $2w_{\mathrm{D}} \sim f \times 10^4$ s^{-1}.

To provide significant momentum transfer from the laser light to the BEC by the coherent mechanism analysed in our present paper, the total radiation exposure time T_{rad} should be of order of the period of the BEC dipole oscillations, at least. So, we accept the following estimation: $T_{\mathrm{rad}} \sim 0.01$ s. After a laser pulse of such a duration has passed, the fraction of atoms still remaining in condensate is equal to $\exp(-2w_{\mathrm{D}}T_{\mathrm{rad}})$. If $f = 0.01$, then the BEC looses more than a half of its initial population. In this case back actions caused by coherent and incoherent processes are of comparable magnitude. But for $f = 0.0001$ depletion is of order of one per cent only, and the BEC interaction with the radiation is essentially coherent. Of course, if the light exposure time is increased, the dissipative processes become more important.

To conclude, we can say that in the present work a self-consistent approach to back action of laser light to a BEC of neutral atoms is developed using two main assumptions: stationarity and negligible light absorption. Study of time-dependent processes including collective oscillation generation in the BEC during switching on the laser and those arising from BEC population losses due to incoherent photon scattering are to become a subject of future works.

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